2017

3rd Semester

MATHEMATICS

PAPER—C6

(Honours)

Full Marks : 40 *Time* : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

1. Answer any *five* questions : 5×1

(a) Let $f : [1, 3] \to \mathbb{R}$ be defined by $f(x) = x^2$. What is $\text{Sup}\{|f(x_1) - f(x_2)| : x_1, x_2 \text{ t } [0, 3]\}$?

(b) Evaluae $\int_0^\infty x^4 e^{-x} dx$.

(c) Find the least positive integer having 24 positive divisors.

(d) Using sine-Cosine form of Beta function, prove that $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$.

(e) If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$, prove that $a \equiv c \pmod{m}$.

(f) If n is an odd positive integer, prove that $\phi(2n) = \phi(n)$.

(g) Find the value of $\sigma(360)$.

(h) State Darboux theorem for the Riemann integral.

2. Answer any *five* questions : 5×2

(a) Show that the function defined by :

$$f(x) = \frac{1}{2^n} \text{ when } \frac{1}{2^{n+1}} < x \le \frac{1}{2n}, n = 0, 1, 2, \ldots\ldots$$

$$= 0 \text{ when } x = 0$$

is integrable in [0, 1].

(b) Examine the convergence of $\int_0^\infty \frac{\cos mx}{x^2 + a^2} dx$, a>0, m>0.

(c) If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b] and $\int_a^b f(x)dx = 0$, prove that there exists at least one point $c \in [a, b]$ such that $f(c) = 0$.

(d) Using Euler's theorem find the unit digit in 3^{100}.

(e) If f in a non-negative continuous function over [0, 1] such that $f\left(\frac{1}{2}\right)>0$, show that $\int_0^1 f(x)dx > 0$.

(f) Examine the convergence of $\int_0^1 \frac{dx}{x^{1/5}(1+x^2)}$.

(g) Show that $\overline{|(n+1)} = n\overline{|(n)}$, for n>0.

(h) If a is prime to b, prove that a^2 is also prime to b.

3. Answer any *three* of the following questions : 3×5

(a) If f is bounded and integrable in [a, b], prove that |f| is bounded and integrable in [a, b]. Is the converse true? Give reason. 3+2

(b) State and prove Dirichlet's theorem for convergence of improper integral. 1+4

(c) Show thtat $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exists if and only if m>0, n>0. 5

(d) State and prove Fermat's theorem. 5

(e) Find the general solution in integers of the equation 5x + 12y = 80. Examine if there is a solution in positive integers. 4+1

4. Answer any *one* question : 1×10

(a) (i) State and prove second mean value theorem of integral calculus in Bonet's form. 1+6

(ii) Prove that $\frac{n^7}{7} + \frac{n^3}{3} + \frac{11n}{21}$ is an integer $\forall n \in \mathbb{N}$. 3

(b) (i) Show that $\lim_{m \to \alpha} \int_0^x \frac{t^m}{t+1}dt = 0$, when $-1<x\leq 1$ 3

(ii) For any positive integer n, prove that $n = \sum_{d/n} \phi(d)$, where the summation extends over all positive divisors of n. 3

(iii) State and prove Wilson's theorem. 1+3

(c) (i) State and prove Chinese remainder theorem. 1+5

(ii) Solve the Diophantine equation 56x + 72y = 40 by Euclidean Algorithm method. 4